

# A Measurement of the $K^0$ Charge Radius and a CP Violating Asymmetry Together with a Search for CP Violating E1 Direct Photon Emission in the Rare Decay

$$K_L \rightarrow \pi^+ \pi^- e^+ e^-$$

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Using the complete KTeV data set of 5241 candidate  $K_L \rightarrow \pi^+ \pi^- e^+ e^-$  decays (including an estimated background of  $204 \pm 14$  events), we have measured the coupling  $g_{CR} = 0.163 \pm 0.014$  (stat)  $\pm 0.023$  (syst) of the CP conserving charge radius process and from it determined a  $K^0$  charge radius of  $\langle r_{K^0}^2 \rangle = (-0.077 \pm 0.007\text{stat} \pm 0.011\text{syst})\text{fm}^2$ . We have also determined a first experimental upper limit of 0.04 (90% CL) for the ratio  $\frac{|g_{E1}|}{|g_{M1}|}$  of the coupling for the E1 direct photon emission process relative to the coupling for M1 direct photon emission process. We also report the measurement of  $|g_{M1}|$  including its associated vector form factor  $|\tilde{g}_{M1}|(1 + \frac{a_1/a_2}{(M_K^2 - M_{K^*}^2) + 2M_K E_{\gamma^*}})$  where  $|\tilde{g}_{M1}| = 1.11 \pm 0.12$  (stat)  $\pm 0.08$  (syst) and  $a_1/a_2 = (-0.744 \pm 0.027$  (stat)  $\pm 0.032$  (syst))  $\text{GeV}^2/c^2$ . In addition, a measurement of the manifestly CP violating asymmetry of magnitude  $(13.6 \pm 1.4$  (stat)  $\pm 1.5$  (syst))% in the CP and T odd angle  $\phi$  between the decay planes of the  $e^+ e^-$  and  $\pi^+ \pi^-$  pairs in the  $K_L$  center of mass system is reported.

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The emission of a virtual photon in the rare decay  $K_L \rightarrow \pi^+ \pi^- e^+ e^-$  proceeds via three main processes: bremsstrahlung, direct photon emission, and the charge radius process. The bremsstrahlung process takes place via the CP violating decay of a  $K_L \rightarrow \pi^+ \pi^-$  followed by emission of an electric dipole (E1) photon by bremsstrahlung from one of the  $\pi$ 's. The direct emission process involves either the CP conserving or CP violating direct emission at the primary decay vertex of a magnetic dipole (M1) or an electric dipole (E1) photon respectively. The CP conserving charge radius process is the transformation of a  $K_L \rightarrow K_S$  by emission of a virtual photon in a J=0 transition (forbidden in real photon emission) followed by the CP conserving decay of the  $K_S$  into  $\pi^+ \pi^-$ . The charge radius coupling is related to the charge radius of the neutral kaon since the virtual photon acts as a probe of the  $K^0$  in a way similar to the virtual

photon in  $K^0$  scattering from an atomic electron. The E1 and M1 direct emission and charge radius couplings are  $g_{E1}$ ,  $g_{M1}$  and  $g_{CR}$ . The matrix elements [1] for the bremsstrahlung, M1, E1 and charge radius processes are

$$\begin{aligned} M_{br} &\sim \eta_{+-} e^{i\delta_0(M_K^2)} \left[ \frac{p_{+\mu}}{p_+ \cdot k} - \frac{p_{-\mu}}{p_- \cdot k} \right] \frac{\bar{u}(k_-) \gamma^\mu v(k_+)}{k^2} \\ M_{M1} &\sim i |g_{M1}| e^{i\delta_1(M_{\pi\pi}^2)} \epsilon_{\mu\nu\rho\sigma} k^\nu p_+^\rho p_-^\sigma \frac{\bar{u}(k_-) \gamma^\mu v(k_+)}{k^2} \quad (1) \\ M_{E1} &\sim |g_{E1}| e^{i(\phi_{+-} + \delta_1(M_{\pi\pi}^2))} [(p_- \cdot k) p_{+\mu} - (p_+ \cdot k) p_{-\mu}] \\ &\quad \cdot \frac{\bar{u}(k_-) \gamma^\mu v(k_+)}{k^2} \end{aligned}$$

$$M_{CR} \sim |g_{CR}| e^{i\delta_0(M_{\pi\pi}^2)} \frac{k^2 P_\mu - (P \cdot k) k_\mu}{M_{\pi\pi}^2 - M_K^2} \frac{\bar{u}(k_-) \gamma^\mu v(k_+)}{k^2}$$

where  $p_+$ ,  $p_-$ ,  $k_+$ ,  $k_-$ ,  $k$ ,  $P$  are the  $\pi^+$ ,  $\pi^-$ , positron,

electron, virtual photon, and  $K_L$  four momenta. The  $\delta_{0,1}$  are the  $I=0,1$   $\pi^+\pi^-$  strong interaction phase shifts.  $\eta_{+-}$  is the coupling of the  $K_L \rightarrow \pi^+\pi^-$  decay.

The KTeV E799-II experiment at Fermi National Accelerator Laboratory previously reported the first observation [2] of the rare four body decay mode  $K_L \rightarrow \pi^+\pi^-e^+e^-$  based on 1% of the KTeV data. We have also made an initial measurement [3] based on 36% of the KTeV  $K_L \rightarrow \pi^+\pi^-e^+e^-$  data of a CP-violating asymmetry in the variable  $\sin\phi\cos\phi$  (where  $\phi$  is the CP- and T-odd angle between the  $e^+e^-$  and  $\pi^+\pi^-$  planes in the  $K_L$  cms). In addition, the measurement of the M1 direct photon emission coupling  $|g_{M1}|$  including a vector form factor was reported in Ref. [3]. In this paper we report a measurement of the charge radius of the  $K^0$  obtained from the coupling  $|g_{CR}|$  of the charge radius process of the  $K_L \rightarrow \pi^+\pi^-e^+e^-$  decay. We also determined an upper limit for the E1 direct photon emission in this decay. Finally, we present the measurements of the M1 direct emission process coupling and its form factor and the CP violating asymmetry in  $\sin\phi\cos\phi$ .

The  $K_L \rightarrow \pi^+\pi^-e^+e^-$  data were accumulated during the 1997 and 1999 runs of the KTeV E799-II experiment. Differences in running conditions and spectrometer configuration can be found in Ref. [4]. The total KTeV E799-II  $K_L \rightarrow \pi^+\pi^-e^+e^-$  signal of 5241 events, including an estimated background of  $204 \pm 14$  events, obtained after the analysis cuts described below, is shown in the  $\pi^+\pi^-e^+e^-$  mass plot of Fig. 1. Note that data has been separately plotted for  $\sin\phi\cos\phi > 0$  and  $\sin\phi\cos\phi < 0$ . The CP violating asymmetry can be seen directly in the mass plot in the differing sizes of the two mass peaks.

The KTeV four track trigger [3] selected  $3.9 \times 10^8$  events from the 97 and 99 runs. Candidate  $K_L \rightarrow \pi^+\pi^-e^+e^-$  events were extracted from these triggers by requiring events to have four tracks that passed track quality cuts and had a common vertex with a good vertex  $\chi^2$ . To be designated as  $e^\pm$ , two of the tracks were required to have opposite charges and  $0.95 \leq E/p \leq 1.05$ , where  $E$  was the energy deposited by the track in the calorimeter, and  $p$  was the momentum obtained from magnetic deflection. To be consistent with a  $\pi^\pm$  pair, the other two tracks were required to have  $E/p \leq 0.90$  and opposite charges. To reduce backgrounds arising from other types of  $K_L$  decays in which decay products have been missed, the candidate  $\pi^+\pi^-e^+e^-$  were required to have transverse momentum  $P_t^2$  of the four tracks relative to the direction of the  $K_L$  be less than  $0.6 \times 10^{-4} \text{ GeV}^2/c^2$ . This cut was 94% efficient for retaining  $K_L \rightarrow \pi^+\pi^-e^+e^-$ .

The major background to the  $K_L \rightarrow \pi^+\pi^-e^+e^-$  mode was  $K_L \rightarrow \pi^+\pi^-\pi_D^0$  where  $\pi_D^0$  was a Dalitz decay,  $\pi^0 \rightarrow \gamma e^+e^-$ , in which the photon was not observed in the CsI calorimeter or the photon vetos. To reduce this background, all  $K_L \rightarrow \pi^+\pi^-e^+e^-$  candidate events were interpreted as  $K_L \rightarrow \pi^+\pi^-\pi_D^0$  decays. Under this

assumption, the longitudinal momentum squared  $(P_L^2)_{\pi^0}$  of the assumed  $\pi^0$  can be calculated in the frame in which the momentum of  $\pi^+\pi^-$  is transverse to the  $K_L$  direction.  $(P_L^2)_{\pi^0}$  was greater than zero for  $K_L \rightarrow \pi^+\pi^-\pi_D^0$  decays except for cases where finite detector resolution resulted in a  $(P_L^2)_{\pi^0} \leq 0$ . In contrast, most of the  $K_L \rightarrow \pi^+\pi^-e^+e^-$  decays had  $(P_L^2)_{\pi^0} \leq 0$ . The requirement that all  $\pi^+\pi^-e^+e^-$  had  $(P_L^2)_{\pi^0} \leq -0.025 \text{ GeV}^2/c^2$  reduced the  $K_L \rightarrow \pi^+\pi^-\pi_D^0$  background under the  $K_L$  peak to 177 events while retaining 94% of the signal.

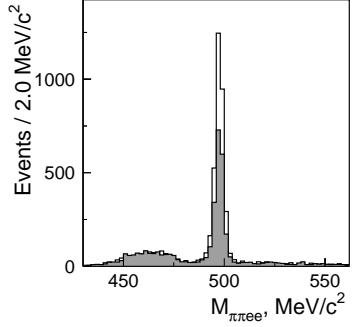


FIG. 1:  $M_{\pi^+\pi^-e^+e^-}$  invariant mass for events passing all  $K_L \rightarrow \pi^+\pi^-e^+e^-$  physics cuts. The superimposed  $K_L$  mass peaks for  $\sin\phi\cos\phi > 0$  (white histogram) and  $< 0$  (gray histogram) directly demonstrate the large CP violating asymmetry. There is no asymmetry in the backgrounds to the two peaks as demonstrated by their complete overlap of the distributions outside the kaon peak region.

A second significant background was due to  $\Xi^0 \rightarrow \Lambda\pi^0 \rightarrow p\pi^-e^+e^-\gamma$  where the photon was missed and the proton was misidentified as a  $\pi^+$ . There were 22 events of background after all cuts due to this decay. All other backgrounds were relatively minor. The largest was due to  $K_L \rightarrow \pi^0\pi^0\pi^0$  with  $\pi^0 \rightarrow e^+e^-e^+e^-$ . This mode contributed approximately four events to the background after cuts. In addition, a potentially large background due to  $K_L \rightarrow \pi^+\pi^-\gamma$  decays in which the photon converted in the material of the spectrometer producing an  $e^+e^-$  pair was eliminated by requiring  $M_{e^+e^-} \geq 2.0 \text{ MeV}/c^2$ . The  $M_{e^+e^-}$  cut retained 95% of the  $K_L \rightarrow \pi^+\pi^-e^+e^-$  events with only one event contributing to the background.

The final requirement of the  $K_L \rightarrow \pi^+\pi^-e^+e^-$  events was that  $492 \text{ MeV}/c^2 \leq M_{\pi\pi ee} \leq 504 \text{ MeV}/c^2$ . The magnitude of the background under the  $K_L$  peak was determined by a fit to the simulated background mass distribution to the wings of the signal region. From this fit, a  $K_L \rightarrow \pi^+\pi^-e^+e^-$  signal of 5037 events above a background of  $204 \pm 14$  events was obtained.

We analyzed the  $K_L \rightarrow \pi^+\pi^-e^+e^-$  decays in a likelihood fit that used the matrix elements (eq. 1) of the model of [1] with additional radiative corrections applied to the final state particles using the PHOTOS pro-

gram [6]. We also found it necessary to include a dependence on the virtual photon energy in the M1 virtual photon emission coupling in order to obtain agreement with the virtual photon energy spectrum  $E_{\gamma^*} = E_{e^+} + E_{e^-}$  of the data (Fig. 2f). The M1 coupling  $|g_{M1}|$  was modified by a form factor

$$|g_{M1}| = |\tilde{g}_{M1}| \left[ 1 + \frac{a_1/a_2}{(M_\rho^2 - M_K^2) + 2M_K E_{\gamma^*}} \right] \quad (2)$$

similar to that used in Ref [7] to describe  $K_L \rightarrow \pi^+ \pi^- \gamma$ . Here  $M_\rho$  is the mass of the  $\rho$  meson (770 MeV/c<sup>2</sup>) and the photon energy has been replaced by  $E_{e^+} + E_{e^-}$ .

The likelihood of a given event (see eq. 3 below), based on the matrix elements  $\mu(\vec{x}_i, \vec{\alpha})$  of the model of Ref. [1], is a function of the five independent variables  $\vec{x}_i$ :  $\phi$ ,  $\theta_{e^+}$  (the angle between the  $e^+$  and the  $\pi^+ \pi^-$  direction in the  $e^+ e^-$  cms),  $\theta_{\pi^+}$  (the angle between the  $\pi^+$  and the  $e^+ e^-$  direction in the  $\pi^+ \pi^-$  cms),  $M_{\pi^+ \pi^-}$ , and  $M_{e^+ e^-}$ . In addition, it depends on the values of the fit parameters  $\vec{\alpha}$ :  $a_1/a_2$  and  $|\tilde{g}_{M1}|$ ,  $|g_{CP}|$  and nominal values for other model parameters such as  $\eta_{+-} = (2.286 \pm 0.017) \times 10^{-3}$  and  $\Phi_{+-} = 43.51^0 \pm 0.06^0$ . The measured strong interaction phase shifts of the  $\pi^+ \pi^-$  were taken from Ref. [8]. The likelihood was calculated using the selected  $K_L \rightarrow \pi^+ \pi^- e^+ e^-$  data sample of  $N_D$  events and a large Monte Carlo event sample  $N_{MC}$  generated with nominal values of the fit parameters  $\vec{\alpha}_0$ , passed through the spectrometer and reconstructed, and then reweighted with a new set of fit parameters  $\vec{\alpha}$  using the matrix elements  $\mu(\vec{x}_i, \vec{\alpha})$ . The likelihood fit to the five independent variables is shown in Fig. 2 along with the fit to  $E_{\gamma^*}$ . The charge radius process contributes to the higher mass  $M_{ee}$  (as shown in the insert of Fig. 2c) while the M1 direct emission is determined by the shape of the  $M_{\pi^+ \pi^-}$  spectrum.

The likelihood function used to perform the fit is

$$\ln L(\vec{\alpha}) = \sum_{i=1}^{N_D} \ln \mu(\vec{x}_i, \vec{\alpha}) - N_D \ln \sum_{j=1}^{N_{MC}} \frac{\mu(\vec{x}_j, \vec{\alpha})}{\mu(\vec{x}_j, \vec{\alpha}_0)} \quad (3)$$

The best fit values were  $a_1/a_2 = (-0.744 \pm 0.027(\text{stat})) \text{ GeV}^2/c^2$ ,  $|\tilde{g}_{M1}| = 1.11 \pm 0.12(\text{stat})$ ,  $|g_{CP}| = 0.163 \pm 0.014(\text{stat})$  and  $\frac{|g_{E1}|}{|g_{M1}|} \leq 0.028$  (upper limit due to statistical uncertainty only). The correlation ( $\rho = 0.924$ ) between  $a_1/a_2$  and  $|\tilde{g}_{M1}|$  has been taken into account in determining their errors.

The distribution of the quantity  $\sin \phi \cos \phi$  (given by  $(\hat{n}_{ee} \times \hat{n}_{\pi\pi}) \cdot \hat{z}(\hat{n}_{ee} \cdot \hat{n}_{\pi\pi})$ , where the  $\hat{n}$ 's are the unit normals to the ee and  $\pi\pi$  planes and  $\hat{z}$  is the unit vector in the  $\pi\pi$  direction in the  $K_L$  cms) is shown in Fig. 2a. The asymmetry of the  $\sin \phi \cos \phi$  distribution

$$A = \frac{N_{\sin \phi \cos \phi > 0.0} - N_{\sin \phi \cos \phi < 0.0}}{N_{\sin \phi \cos \phi > 0.0} + N_{\sin \phi \cos \phi < 0.0}} \quad (4)$$

yields  $(23.8 \pm 1.4 \text{ (stat)})\%$  before acceptance corrections. Using the fit of the model of Ref. [1] to the data to determine the acceptance, an asymmetry integrated over the entire  $K_L \rightarrow \pi^+ \pi^- e^+ e^-$  phase space of  $(13.6 \pm 1.4 \text{ (stat)})\%$  was obtained, the largest such CP violating effect yet observed in kaon decay. The interference between the M1 direct emission process and the bremsstrahlung process generates the asymmetry in the  $\sin \phi \cos \phi$  distribution.

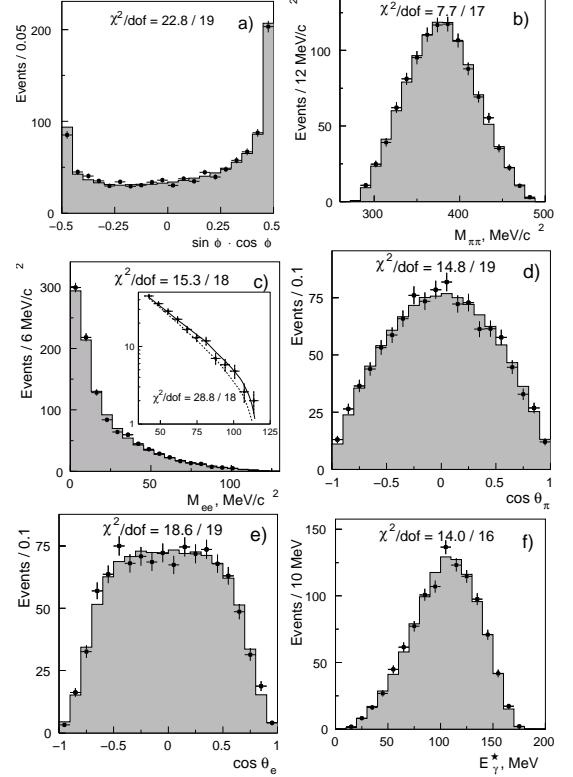


FIG. 2: Likelihood fit to the five independent variables a)  $\sin \phi \cos \phi$ , b)  $M_{\pi^+ \pi^-}$ , c)  $M_{e^+ e^-}$  (the dotted curve in the insert in this figure shows the deficit of  $e^+ e^-$  pairs at high  $M_{ee}$  if the charge radius process is ignored. The  $\chi^2/\text{dof}$  for the fit of the model to the data increases from 0.85 to 1.6 if the charge radius process is left out.), d)  $\theta_{\pi^+}$ , and e)  $\theta_{e^+}$  of the  $K_L \rightarrow \pi^+ \pi^- e^+ e^-$  decay; f)  $E_{\gamma^*}$  defined as  $E_{e^+} + E_{e^-}$

Possible sources of false asymmetries were considered including those due to backgrounds and asymmetries in the detector. To check for detector asymmetries, a sample of  $15 \times 10^6$   $K_L \rightarrow \pi^+ \pi^- \pi_D^0$  decays, which are expected to have no  $\phi$  asymmetry and which have similar topology to  $K_L \rightarrow \pi^+ \pi^- e^+ e^-$  except for the presence of an extra photon in the CsI, was examined, and an asymmetry of  $((3.3 \pm 2.6) \times 10^{-2})\%$  was measured.

Systematic errors on  $a_1/a_2$ ,  $|\tilde{g}_{M1}|$ ,  $|g_{CR}|$  and  $\frac{|g_{E1}|}{|g_{M1}|}$  due to several sources are shown in Table I below. As shown in Table I, the dominant systematic error is due to the variation of the fitted parameters resulting from varying the physics cuts used to select the  $K_L \rightarrow \pi^+ \pi^- e^+ e^-$  data

and Monte Carlo events and repeating the fit procedure. Some analysis cut variations significantly increased the level of backgrounds to the  $K_L \rightarrow \pi^+\pi^-e^+e^-$  mass peak. These cuts were separated from the other physics cuts and are labeled as “background” in Table I. Finally, input parameters to the Monte Carlo such as  $\eta_{+-}, \Phi_{+-}$ , and the strong interaction  $\pi^+\pi^-$  phases shifts  $\delta_{0,1}$  that were not included in the likelihood fit were varied by  $\pm 1\sigma$  of their published values to determine the uncertainty in the fit parameters due to their uncertainties. The total systematic errors in  $a_1/a_2$ ,  $|\tilde{g}_{M1}|$ ,  $|g_{CR}|$  and  $\frac{|g_{E1}|}{|g_{M1}|}$  were obtained by adding the systematic errors in quadrature.

The systematic errors in the  $\phi$  asymmetry due to several sources are given in Table II below. The physics cut variations and background systematics of the  $\phi$  angle asymmetry have been determined as discussed above. The  $\eta_{+-}$ ,  $\Phi_{+-}$  and  $\delta_{0,1}$  systematics are obtained as before using the  $\pm 1\sigma$  uncertainties in these parameters. Additional uncertainties of the asymmetry due to the one  $\sigma$  uncertainties of the fitted parameters are also included. All systematic errors of Table II are added in quadrature to obtain the total systematic error.

In conclusion, the KTeV collaboration measured a charge radius coupling  $|g_{CP}| = 0.163 \pm 0.014(\text{stat}) \pm 0.023(\text{syst})$  which has been used to obtain, in a novel way [1], a  $K^0$  charge radius of  $\langle r_{K^0}^2 \rangle = -3|g_{CR}|/M_K^2 = (-0.077 \pm 0.007(\text{stat}) \pm 0.011(\text{syst})) (fm^2)$ , consistent with the previous measurements of the  $K^0$  charge radius [9, 10, 11] obtained in  $K^0$  electron scattering and from a similar analysis of the  $K_L \rightarrow \pi^+\pi^-e^+e^-$  mode by NA48 [12]. We also set a first experimental upper limit on the presence of E1 direct photon emission in the  $K_L \rightarrow \pi^+\pi^-e^+e^-$  mode of  $\frac{|g_{E1}|}{|g_{M1}|} < 0.04$  (90%CL) including systematic errors. In addition, the M1 photon emission coupling was measured to be  $|\tilde{g}_{M1}| = 1.11 \pm 0.12$  (stat)  $\pm 0.08$  (syst) plus a vector form factor as given in equation (2) with  $a_1/a_2 = (-0.744 \pm 0.027)$  (stat)  $\pm 0.032$  (syst)  $GeV^2/c^2$ . Using  $a_1/a_2$  and  $|\tilde{g}_{M1}|$ , an average  $|g_{M1}|$  over the range of  $E_{\gamma^*}$  was calculated to be  $0.74 \pm 0.04$ . Finally, we made a measurement of a large CP-violating asymmetry in the distribution of T-odd angle  $\phi$  in  $K_L \rightarrow \pi^+\pi^-e^+e^-$  decays of  $(13.6 \pm 1.4 \text{ (stat)} \pm 1.5 \text{ (syst)})\%$  consistent with the theoretically expected asymmetry of Refs. [1, 5]. This result is consistent with our original measurement [3] and a later measurement by NA48 [12].

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Source	Uncertainty in			
	$a_1/a_2$	$ \tilde{g}_{M1} $	$ g_{CR} $	$\frac{ g_{E1} }{ g_{M1} }$
Monte Carlo Statistics	0.002	0.01	0.001	0.001
Choice of initial MC parameters	0.005	0.02	0.001	0.001
Skewing from input MC values	0.000	0.028	0.002	0.010
Physics cut variations	0.022	0.041	0.021	0.018
Background	0.022	0.05	0.01	0.008
$\eta_{+-}$ Uncertainty	0.0001	0.01	0.002	0.0002
$\Phi_{+-}$ Uncertainty	0.0003	0.002	0.0002	0.0005
$\delta_{0,1}$ Uncertainty	0.001	0.004	0.001	0.0003
Total Systematic Error	0.032	0.08	0.023	0.023

TABLE I: Syst. errors of  $a_1/a_2$ ,  $|\tilde{g}_{M1}|$ ,  $|g_{CR}|$  and  $\frac{|g_{E1}|}{|g_{M1}|}$

Source	$\Delta$ Asymmetry (%)
Physics cut variations	0.71
Background	0.30
$\eta_{+-}$ Uncertainty	0.163
$\Phi_{+-}$ Uncertainty	0.111
$\delta_{0,1}$ Uncertainty	0.325
$\frac{ g_{E1} }{ g_{M1} }$	0.326
$ g_{M1} , a_1/a_2$	0.335
$ g_{CR} $	0.335
Total Systematic Error	1.46

TABLE II: CP violating  $\phi$  asymmetry systematic errors

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